Queueing Network and Some Types of Customers and Signals

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Joint work with J.M Fourneau, J. Mairesse, M.A Tran

VMS-SMF Joint Congress, Hue, August 23^{th} 2012

Plan

1. Introduction-Preliminaries
   - Queue? Network?
2. 0-automatic queues and networks
   - Introduction of 0-automatic queues
   - Results on 0-automatic queues and networks
3. Some new types of signals in G-networks

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In daily life

Mathematical study

A.K. Erlang *(1909)*: queue of telephones

Kendall nomenclature *(1953)*:
$A/S/n/K/D$

- A : inter-arrival time distribution
- S : service time distribution
- n : number of servers
- K : capacity of the buffer
- D : discipline of service
  First In First Out,
  LIFO,
  PS, . . .

Mathematical study

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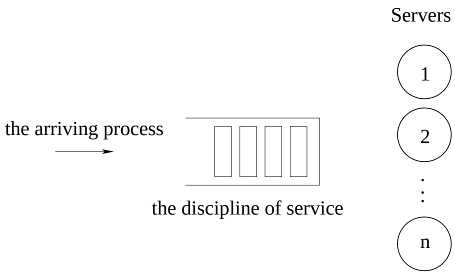

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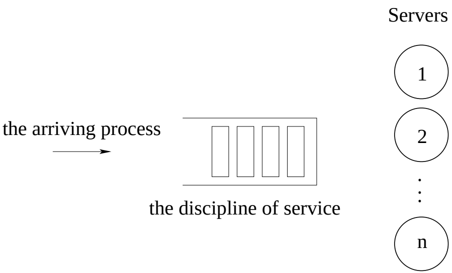

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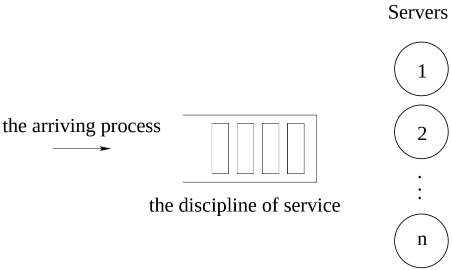

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Queues can be connected to form queueing networks

Jackson network
Probabilistic routing

Kelly network
fixed routing

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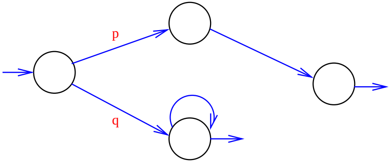

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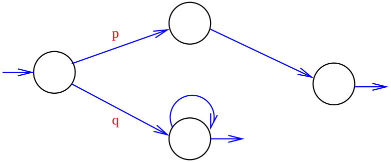

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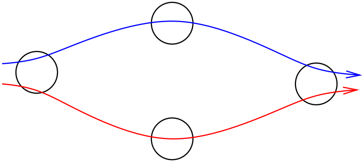
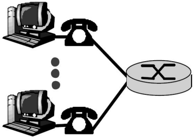
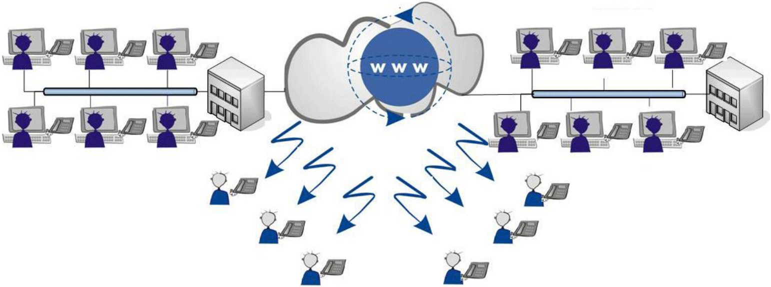


Application





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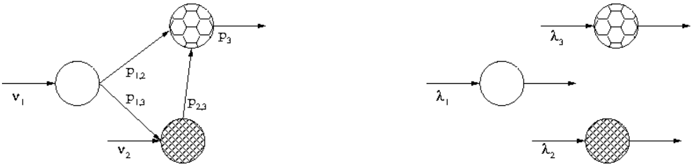

Product form: a network $\Longleftrightarrow$ independent queues

Jackson network *(1957)*, BCMP network *(Baskett et al, 1975)*, Kelly network *(1979)*, G-network *(Gelenbe, 1989)*

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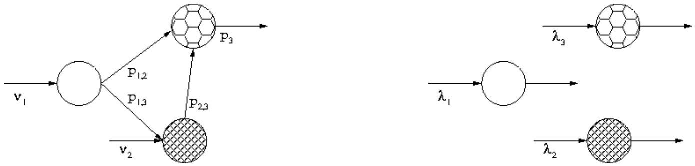

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0-automatic network *(Dao Thi and Mairesse)*

Network with signals

- 90s: Negative customer *(Gelenbe)*
- Some types of signals:
  - Reset, catastrophe, batch *(Gelenbe, 1993, 2002, Chao, 1995)*
  - Negative signal, positive signal *(Chao et al, 1999)*
- Service time: exponential, Cox
- Recent results *(Dao Thi, Fourneau and Tran, 2010, 2011, 2012)*:
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Two first examples of (0-automatic) queues

The simple queue $M/M/1/\infty/FIFO$

- Buffer content: $n \in \mathbb{N}$
- An arrival: $n \to n + 1$
- Stability condition: $\lambda < \mu$
- Stationary distribution:

$$\pi(n) = (1 - \frac{\lambda}{\mu})(\frac{\lambda}{\mu})^n$$

- Burke theorem: departure process is Poisson of rate λ

$\to$ a trivial random walk (r.w.) on $(\mathbb{N}, +)$, jumps +1.

Gelenbe 's G-queue with positive and negative customers

- 2 types of customers: $\{1\}$, $\{-1\}$. Buffer content: $n \in \mathbb{Z}$
- $\{1\}$-customer: $n \to n + 1$, $\{-1\}$-customer: $n \to n - 1$
- ν: probability measure on $\{-1, 1\}$.

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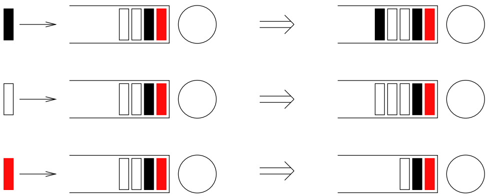
Combine 2 models

- Set of possible types: $\Sigma = \{a, b, b^{-1}\}$.
- Buffer content: a word in

$$L = \left\{ u_k \cdots u_1 \in \Sigma^* \mid \forall i,\ u_{i+1}u_i \notin \{bb^{-1}, b^{-1}b\} \right\} . \qquad (1)$$

Customers of type a, b, b^{-1} are resp. in black, red and white.



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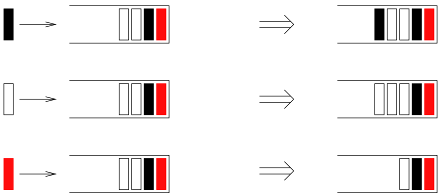

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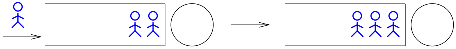
Classical type.



Positive/negative type.

"One equals many" type.

"Dating agency" type.

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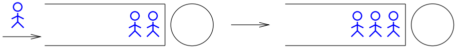

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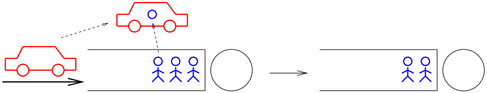

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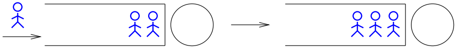

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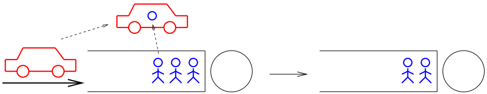

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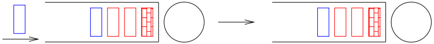

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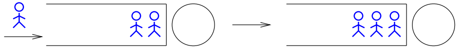

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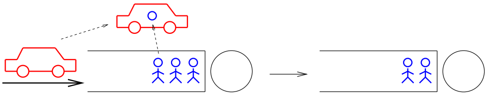

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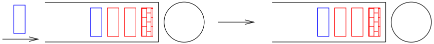

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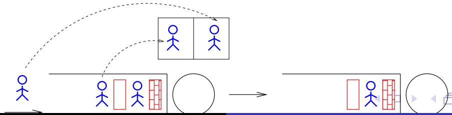

Result for 0-automatic queues

For FIFO 0-automatic queues:

- Stationary distribution and stationary condition
- Burke theorem for departure process: Poisson

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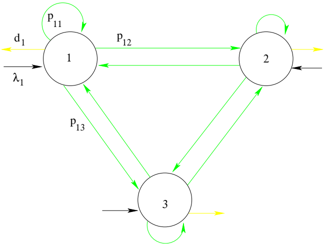
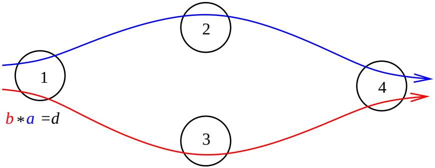


Kelly-like network : fixed routing
Fusion case $b * a = d$:



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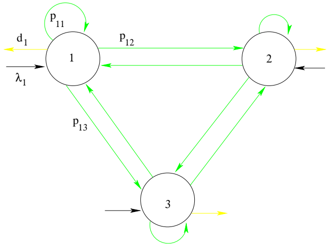

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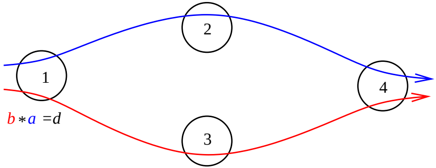

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Positive signal *(X.Chao et al, 1999)* : add a customer to a queue

Synchronised arrivals: a signal will add one customer to some queues in the network
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Group-deletion signals: delete all customer of the same class at the back-end of the buffer
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Change-class signal and change phase signal

Change-class Signal:

- Signal $S_{a,b}$: a-customer $\hookrightarrow$ b-customer
- $S_{a,b} * a = b$ $\longrightarrow$ link with 0-automatic mechanism
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Change-phase Signal:

- Phase-type service time
- Changing-phase signal: skip a phase of service
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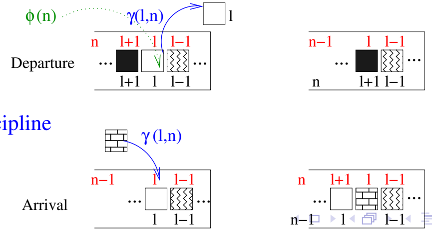


T H A N K
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